Relativistic quantum corrections to laser wakefield acceleration

Jun Zhu
Department of Physics, Shanghai University, Shanghai 200444, China

Peiyong Ji*
Department of Physics, Shanghai University, Shanghai 200444, China
and The Shanghai Key Laboratory of Astrophysics, Shanghai 200234, China

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The influence of quantum effects on the interaction of intense laser fields with plasmas is investigated by using a hydrodynamic model based on the framework of the relativistic quantum theory. Starting from the covariant Wigner function and Dirac equation, the hydrodynamic equations for relativistic quantum plasmas are derived. Based on the relativistic quantum hydrodynamic equations and Poisson equation, the perturbations of electron number densities and the electric field of the laser wakefield containing quantum effects are deduced. It is found that the corrections generated by the quantum effects to the perturbations of electron number densities and the accelerating field of the laser wakefield cannot be neglected. Quantum effects will suppress laser wakefields, which is a classical manifestation of quantum decoherence effects, however, the contribution of quantum effects for the laser wakefield correction will been partially counteracted by the relativistic effects. The analysis also reveals that quantum effects enlarge the effective frequencies of plasmas, and the quantum behavior appears a screening effect for plasma electrons.

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I. INTRODUCTION

The interaction of intense laser fields with plasmas has received much theoretical and experimental attention during the last decades. With the development of ultrashort pulse laser technology, a large-amplitude relativistic plasma wave is excited by the ponderomotive force from ultraintense laser pulses and it can generate such an extremely high accelerating field of 100 GV/cm, which means that plasma electrons can be accelerated to 10 GeV over 1 mm distances [1–4]. The relativistic laser wakefields may become the next generation of particle accelerators. In 2004, four international laboratories reported that they obtained monoenergetic electron beams by laser wakefield acceleration, which have provided an actual practicable prospect of high-energy electron beams accelerated by the laser wakefields.

Traditional plasma physics has mainly been limited in the regimes dominated by classical physics, in which quantum-mechanical effects are neglected. However, In some plasmas under extreme physical conditions, such as in ultrasmall electronic devices [5], dense astrophysical plasmas [6–9], and some laser plasmas [10], quantum effects have to be taken into account and there is much attention to practical applications of quantum mechanics in plasma physics where the quantum nature of particles plays a critical role.

As a new emerging field in plasma physics, quantum plasmas have received extensive attention and interests. The quantum effects become important in dense plasmas, when the electron thermal de Broglie wavelength approaches the electron Fermi wavelength λ_F and exceeds the electron Debye radius λ_D (viz., $\lambda_B \sim \lambda_F > \lambda_D$) [11,12]. Recent researches involve quantum ion-acoustic waves [13], quantum drift waves [14], and modifications in Debye screening of quantum plasmas [15]. In addition, Manfredi has reviewed different approaches to the modeling of quantum effects in collisionless plasmas [16]. A new dispersion relationship for electromagnetic drift modes in a nonuniform cold plasma was obtained by Shukla and Ali [17]. Moreover, relativistic quantum plasmas have been also investigated in the last decades. Using Wigner function approach, electromagnetic perturbations [18], quantum fluctuations [19], and relativistic quantum gas [20] were researched. The dispersion function of relativistic quantum plasmas was derived by Melrose in 2006 [21].

In the interaction of intense laser fields with plasmas, especially in the laser wakefield accelerators, quantum effects should be taken into account. It is noticed that plasma electrons will become relativistic under the action of intense laser fields, and traditional hydrodynamic models for nonrelativistic plasmas cannot be used to describe the physical process of laser wakefield acceleration. In this paper, by using a relativistic quantum hydrodynamic model, we have studied the relativistic quantum effects in the processes of the interaction of intense laser fields with plasmas. Based on the relativistic quantum hydrodynamic equations and the Poisson equation, the perturbation of electron number density and the laser wakefields including quantum effects are derived. By comparing the corrected results with the classical ones, it is found that quantum effects weaken the laser wakefields.

This paper is organized as follows. In Sec. II, the hydrodynamic equations for relativistic quantum plasmas are derived starting from the covariant Wigner function and Dirac equation. In Sec. III, the perturbations of electron number densities and the laser wakefields considering quantum effects are derived and the contribution of quantum effects to the laser wakefields is calculated. In Sec. IV, the discussion and conclusion for quantum effects are presented.

*pyji@staff.shu.edu.cn

II. HYDRODYNAMIC MODEL FOR RELATIVISTIC QUANTUM PLASMAS

A magnetohydrodynamic model for quantum plasmas was put forward by Haas in 2005 [22]. Since starting from the nonrelativistic quantum equations (Wigner function and Schrödinger equation), the magnetohydrodynamic model established by Haas is nonrelativistic. Under the action of intense laser fields, however, plasma electrons become relativistic and the nonrelativistic hydrodynamic model is no longer valid, so the relativistic hydrodynamic model for quantum plasmas should be established to deal with the interaction of intense laser pulses with plasmas.

It is well known that Wigner function is a kind of quantum mechanical distribution function, and it can be used to calculate some average values of observable. In the relativistic case, the Wigner distribution is no more unique than it is in the classical case. In this paper, the covariant one-particle Wigner distribution under the external electromagnetic fields is defined as [23,24] by

$$f^{\lambda}(x,p) = \frac{1}{(2\pi\hbar)^4}\int d^4R \exp(-i\pi\cdot R/\hbar) \times\left\langle \bar{\psi}\left(x+\frac{1}{2}R\right)\gamma^{\lambda}\psi\left(x-\frac{1}{2}R\right)\right\rangle, \quad (1)$$

where $\pi^{\mu}=p^{\mu}+eA^{\mu}/c$, ψ and $\bar{\psi}$ are Dirac's fields obeying

$$\left[\gamma^{\mu}\left(i\hbar\partial_{\mu}-\frac{e}{c}A_{\mu}\right)-mc\right]\psi(x)=0, \quad (2)$$

and the bracket $\langle\ldots\rangle$ is a quantum statistical average defined as

$$\langle\hat{A}\rangle \equiv \mathrm{Tr}\{\rho\hat{A}\}, \quad (3)$$

where $\hat{A}$ is any operator and ρ is the density operator denoting the statistical state of a system. In the above equations, the metric has the signature $+---$, and $e=-|e|$.

Taking derivative of Eq. (1) and using the Dirac Eq. (2), a kinetic equation satisfied by $f^{\lambda}(x,p)$ is obtained as

$$\partial_{\lambda}f^{\lambda}(x,p)+\frac{e}{c}F^{\mu}_{\lambda}\frac{\partial}{\partial p^{\mu}}f^{\lambda}(x,p)=0. \quad (4)$$

By using the algebra of γ matrices $\{\gamma^{\mu},\gamma^{\nu}\}=2\eta^{\mu\nu}$, the Dirac current operator in Eq. (1) can be decomposed into two parts as

$$\bar{\psi}\left(x+\frac{1}{2}R\right)\gamma^{\lambda}\psi\left(x-\frac{1}{2}R\right) = \frac{i\hbar}{2mc}\bar{\psi}\left(x+\frac{1}{2}R\right)\overleftrightarrow{\partial}^{\lambda}\psi\left(x-\frac{1}{2}R\right) - \frac{e}{2mc^2}\left[A^{\lambda}\left(x+\frac{1}{2}R\right)+A^{\lambda}\left(x-\frac{1}{2}R\right)\right]\bar{\psi}\left(x+\frac{1}{2}R\right)\psi\left(x-\frac{1}{2}R\right) - \frac{e}{2mc^2}\left[A_{\mu}\left(x+\frac{1}{2}R\right)-A_{\mu}\left(x-\frac{1}{2}R\right)\right]\bar{\psi}\left(x+\frac{1}{2}R\right)\sigma^{\mu\lambda}\psi\left(x-\frac{1}{2}R\right) - \frac{i\hbar}{2mc}\partial_{\mu} \times\left[\bar{\psi}\left(x+\frac{1}{2}R\right)\sigma^{\mu\lambda}\psi\left(x-\frac{1}{2}R\right)\right], \quad (5)$$

where $\varphi\overleftrightarrow{\partial}^{\lambda}\psi=\varphi\partial^{\lambda}\psi-(\partial^{\lambda}\varphi)\psi$ and $\sigma^{\mu\lambda}=(\gamma^{\mu}\gamma^{\lambda}-\gamma^{\lambda}\gamma^{\mu})/2$, the first and second terms on the right side of Eq. (5) are the convective part of the current, and the third and forth terms are spin contribution to the current. In the weak relativistic approximation, the spin contribution in Eq. (5) can be neglected and it provides

$$f^{\lambda}(x,p) \simeq \frac{p^{\lambda}}{mc}f(x,p), \quad (6)$$

and Eq. (4) can be rewritten by

$$p^{\lambda}\partial_{\lambda}f(x,p)+\frac{e}{c}F^{\mu}_{\lambda}p^{\lambda}\frac{\partial}{\partial p^{\mu}}f(x,p)=0, \quad (7)$$

where

$$f(x,p) = \frac{1}{(2\pi\hbar)^4}\int d^4R \exp(-i\pi\cdot R/\hbar) \times\left\langle \bar{\psi}\left(x+\frac{1}{2}R\right)\psi\left(x-\frac{1}{2}R\right)\right\rangle. \quad (8)$$

It should be noted that Eq. (7) is still a quantum equation: $f(x,p)$ is not positive definite. In order to derive a fluid model, we introduce the definitions of four-current and momentum-energy tensor

$$J^{\lambda}(x) = \frac{e}{m}\int d^4p\,p^{\lambda}f(x,p), \quad (9)$$

and

$$T^{\nu\lambda} = \frac{1}{m}\int d^4p p^\nu p^\lambda f(x,p). \tag{10}$$

Taking moments of Eq. (7) and using the definitions (9) and (10), the covariant forms of the relativistic quantum hydrodynamic equations are obtained as

$$\partial_\lambda J^\lambda = 0, \tag{11}$$

and

$$\partial_\lambda T^{\nu\lambda} = \frac{1}{c}F^\nu_\lambda J^\lambda. \tag{12}$$

By introducing the momentum-energy tensor $T^{\nu\lambda}$ of the perfect fluids

$$T^{\nu\lambda} = -P\eta^{\nu\lambda} + \left(\frac{P}{c^2} + mn\right)U^\nu U^\lambda, \tag{13}$$

where P and $U^\nu = (\gamma c, \gamma \mathbf{u})$ are the pressure and four-dimensional velocity of the electron fluids, respectively, with $\gamma = 1/\sqrt{1-(u/c)^2}$ is the relativistic factor. Equations (11) and (12) can be expressed in three-dimensional vector forms

$$\partial_t(\gamma n) + \nabla\cdot(\gamma n \mathbf{u}) = 0, \tag{14}$$

and

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = \frac{enc^2}{(P+mnc^2)\gamma}\left(\mathbf{E} + \frac{\mathbf{u}}{c}\times\mathbf{B} - \frac{1}{c^2}\mathbf{u}\mathbf{u}\cdot\mathbf{E}\right) - \frac{c^2}{(P+mnc^2)\gamma^2}\left(\nabla P + \frac{\boldsymbol{\beta}}{c}\frac{\partial P}{\partial t}\right). \tag{15}$$

It immediately notices that Eq. (15) do not differ from the hydrodynamic equations for classical relativistic plasmas. This may seem surprising, but in the following it will appear that the quantum nature of this system is in fact hidden in the pressure term. The pressure term may be decomposed into a classical and a quantum part, which will be shown as follows.

Using the previous expressions (9), (10), and (13), one can compute the pressure. After some algebraic calculations and under the weak relativistic approximation $P \ll mnc^2$, one obtains

$$\begin{aligned} P\eta^{\nu\lambda} = {} & \frac{1}{mn}\left[\frac{i\hbar}{2}\langle\bar{\psi}(x)\overleftrightarrow{\partial}^\nu\psi(x)\rangle - \frac{e}{c}A^\nu(x)\langle\bar{\psi}(x)\psi(x)\rangle\right] \\ & \times\left[\frac{i\hbar}{2}\langle\bar{\psi}(x)\overleftrightarrow{\partial}^\lambda\psi(x)\rangle - \frac{e}{c}A^\lambda(x)\langle\bar{\psi}(x)\psi(x)\rangle\right] \\ & - \frac{e^2}{mc^2}A^\nu(x)A^\lambda(x)\langle\bar{\psi}(x)\psi(x)\rangle + \frac{i\hbar e}{2mc}A^\nu(x) \\ & \times\langle\bar{\psi}(x)\overleftrightarrow{\partial}^\lambda\psi(x)\rangle + \frac{i\hbar e}{2mc}A^\lambda(x)\langle\bar{\psi}(x)\overleftrightarrow{\partial}^\nu\psi(x)\rangle \\ & + \frac{\hbar^2}{4m}\langle[\partial^\nu\bar{\psi}(x)][\partial^\lambda\psi(x)] + [\partial^\lambda\bar{\psi}(x)][\partial^\nu\psi(x)] \\ & - [\partial^\nu\partial^\lambda\bar{\psi}(x)]\psi(x) - \bar{\psi}(x)\partial^\nu\partial^\lambda\psi(x)\rangle. \end{aligned} \tag{16}$$

If we introduce the decomposition of spinor as

$$\psi = \sqrt{n}\exp(iS/\hbar)\varphi, \tag{17}$$

where φ is the two-spinor carrying the spin $\frac{1}{2}$ properties, which satisfies $\varphi^\dagger\varphi = 1$, we obtain $P = P^C + P^Q$, where the classical P^C and quantum P^Q parts of the pressure are

$$P^C = \frac{n}{m}[\langle(\nabla S)^2\rangle - \langle(\nabla S)\rangle^2], \tag{18}$$

and

$$P^Q = \frac{\hbar^2}{2m}[(\nabla\sqrt{n})^2 - \sqrt{n}\nabla^2\sqrt{n}]. \tag{19}$$

Using the approximation relation $\mathbf{p} \simeq \nabla S$, where $\mathbf{p}$ is the electron momentum of the plasmas, the classical pressure can be represented

$$P^C = \frac{n}{m}[\langle\mathbf{p}^2\rangle - \langle\mathbf{p}\rangle^2], \tag{20}$$

which is a standard pressure originated the momentum fluctuations of the particles in plasmas. Under the low temperatures and high-densities plasma conditions, the quantum nature of particles plays a critical role. Inserting the expression of quantum pressure into Eq. (15) and neglecting the contribution of the classical pressure, the hydrodynamic equation for relativistic quantum plasmas is obtained as

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = {} & \frac{e}{m\gamma}\left(\mathbf{E} + \frac{\mathbf{u}}{c}\times\mathbf{B} - \frac{1}{c^2}\mathbf{u}\mathbf{u}\cdot\mathbf{E}\right) \\ & + \frac{\hbar^2}{2m^2\gamma^2}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) - \frac{\hbar^2\boldsymbol{\beta}}{2m^2\gamma^2 c}\partial_t[(\nabla\sqrt{n})^2 \\ & - \sqrt{n}\nabla^2\sqrt{n}]. \end{aligned} \tag{21}$$

The second term on the right side of Eq. (21) is the Bohm potential with the correction of relativistic factor and the third term is the coupling correction of relativistic and quantum effects. If one sets $\hbar$ equal to zero, the classical relativistic hydrodynamic equation is recovered.

III. RELATIVISTIC QUANTUM CORRECTION TO LASER WAKEFIELD ACCELERATION

With the development of ultrashort pulse laser technology, a large-amplitude relativistic plasma wave is excited by the ponderomotive force from ultraintense laser pulses and it can generate an extremely high-accelerating field. The relativistic laser wakefields may become the next generation of particle accelerators. When a plasma is characterized under the regimes of low temperatures and high densities, which means that de Broglie wavelength of electrons is similar to the average interparticle distances in plasmas, quantum effects will play a significant role. In this section, we will investigate the quantum effects in the physical processes of the laser wakefield acceleration.

A laser field is specified by the vector potential $\mathbf{A}_L$ along with the gauge condition $\nabla\cdot\mathbf{A}_L = 0$. The ponderomotive force, exerted by the laser pulse on the plasma, is given by $F_{pond} = |e|\nabla\Phi_L$, where the ponderomotive potential is $\Phi_L =$

$-mc^2 a_L/(2|e|)$, and $\mathbf{a}_L = |e|\mathbf{A}_L/(mc^2)$ is the normalized vector potential of the radiation field.

It is assumed that under the action of the laser fields, the perturbation of the plasma is only generated from the movement of electrons and the ions remain as a stationary and homogeneous background. The plasma electrons obey the following relativistic quantum hydrodynamic equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = \frac{e}{m\gamma}\left(\mathbf{E} + \frac{\mathbf{u}}{c}\times\mathbf{B} - \frac{1}{c^2}\mathbf{u}\mathbf{u}\cdot\mathbf{E}\right) + \frac{\hbar^2}{2m^2\gamma^2}\left\{\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) - \frac{\boldsymbol{\beta}}{c}\partial_t[(\nabla\sqrt{n})^2 - \sqrt{n}\nabla^2\sqrt{n}]\right\}, \tag{22}$$

$$\frac{\partial}{\partial t}(\gamma n) + \nabla\cdot(\gamma n\mathbf{u}) = 0, \tag{23}$$

and

$$\nabla\cdot\mathbf{E} = -4\pi|e|(n - n_0), \tag{24}$$

where n_0 and n are the plasma electron densities before and after the driving laser beam injecting into the plasmas. Under the weak relativistic condition $|\mathbf{a}_L|^2 \ll 1$, Eqs. (22)–(24) can be expanded to one order in a_L,

$$\frac{\partial}{\partial t}\mathbf{u}^{(1)} = c\frac{\partial}{\partial t}\mathbf{a}_L, \tag{25}$$

$$\frac{\partial}{\partial t}n^{(1)} + n_0\nabla\cdot\mathbf{u}^{(1)} = 0, \tag{26}$$

and

$$\nabla^2\phi^{(1)} = 4\pi|e|n^{(1)}, \tag{27}$$

where ϕ is the scalar potential of the electromagnetic field and $\mathbf{u}^{(1)}$ is just the quiver velocity of the electrons in the laser field. The solutions of the first-order equations are $\mathbf{u}^{(1)} = c\mathbf{a}_L$, $n^{(1)} = \phi^{(1)} = 0$. To the second order in a_L, Eqs. (22)–(24) give

$$\frac{\partial \mathbf{u}^{(2)}}{\partial t} = \frac{|e|}{m}\nabla[\phi^{(2)} + \Phi_L] + \frac{\hbar^2}{2m^2}\left\{\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) - \frac{\boldsymbol{\beta}}{c}\partial_t[(\nabla\sqrt{n})^2 - \sqrt{n}\nabla^2\sqrt{n}]\right\}, \tag{28}$$

$$\frac{\partial}{\partial t}n^{(2)} + n_0\nabla\cdot\mathbf{u}^{(2)} = 0, \tag{29}$$

and

$$\nabla^2\phi^{(2)} = 4\pi|e|n^{(2)}. \tag{30}$$

The first term on the right of Eq. (28) represents the restoring electrostatic force of the plasma, whereas the second term represents the outward ponderomotive force of the laser pulse and the last term represents the quantum correction to the momentum equation of electrons. Relativistic effects do not enter the plasma response until the third order, so we don't consider the relativistic factor γ.

From Eqs. (28)–(30), the density response and electrostatic potential of the plasma are deduced as

$$(\partial_t^2 + \omega_{p0}^2)\frac{n^{(2)}}{n_0} = \frac{c^2}{2}\nabla^2 a_L^2 - \frac{\hbar^2}{4m^2 n_0}\left[\nabla^2\nabla^2 n^{(2)} + \frac{\boldsymbol{\beta}}{c}\partial_t\nabla\nabla^2 n^{(2)}\right], \tag{31}$$

and

$$(\partial_t^2 + \omega_{p0}^2)\phi^{(2)} = -\omega_{p0}^2\Phi_L - \frac{\pi|e|\hbar^2}{m^2}\left[\nabla^2 n^{(2)} + \frac{\boldsymbol{\beta}}{c}\partial_t\nabla n^{(2)}\right]. \tag{32}$$

Take the approximation of $n^{(2)}(\mathbf{r},t) = n^{(2)}\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$ and $\phi^{(2)}(\mathbf{r},t) = \phi^{(2)}\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$, Eqs. (31) and (32) can be rewritten as

$$(\partial_t^2 + \omega_{p0}^2)\frac{n^{(2)}}{n_0} = \frac{c^2}{2}\nabla^2 a_L^2 - (1-\beta)\frac{k^4}{4k_c^2 k_{p0}^2}\omega_{p0}^2\frac{n^{(2)}}{n_0}, \tag{33}$$

and

$$(\partial_t^2 + \omega_{p0}^2)\phi^{(2)} = -\omega_{p0}^2\Phi_L - (1-\beta)\frac{k^4}{4k_c^2 k_{p0}^2}\omega_{p0}^2\phi^{(2)}, \tag{34}$$

where $k_c = mc/\hbar$ is the Compton wave number of electron, and $k_{p0} = \omega_{p0}/c$. Introducing the effective plasma frequency $\tilde{\omega}_{p0}^2 = (1+\Delta)\omega_{p0}^2$, Eqs. (33) and (34) are transformed to

$$(\partial_t^2 + \tilde{\omega}_{p0}^2)\frac{n^{(2)}}{n_0} = \frac{c^2}{2}\nabla^2 a_L^2, \tag{35}$$

and

$$(\partial_t^2 + \tilde{\omega}_{p0}^2)\phi^{(2)} = -\omega_{p0}^2\Phi_L, \tag{36}$$

where $\Delta = (1-\beta)k^4/4k_c^2 k_{p0}^2$ is the relativistic quantum corrected term. To solve Eqs. (35) and (36), it is convenient to perform a transformation of the speed of light frame $\xi = z - ct$, $\tau = t$. A temporal steady state, $\partial/\partial\tau = 0$, in the laser pulse frame is assumed.

The solutions of Eqs. (35) and (36) read

$$\frac{n^{(2)}(r,\xi)}{n_0} = \frac{1}{2\tilde{k}_{p0}}\int_{\xi_0}^{\xi} d\xi' \sin\tilde{k}_{p0}(\xi - \xi')\nabla^2 a_L^2, \tag{37}$$

and

$$\phi^{(2)}(r,\xi) = -\frac{k_{p0}^2}{\tilde{k}_{p0}}\int_{\xi_0}^{\xi} d\xi' \sin\tilde{k}_{p0}(\xi - \xi')\Phi_L(r,\xi'), \tag{38}$$

where $\tilde{k}_{p0} = \tilde{\omega}_{p0}/c$. The axial and radial wakefields are then given by $E_z = -\partial\phi/\partial\xi$ and $E_r = -\partial\phi/\partial r$, respectively, and it can further be shown that $\partial E_z/\partial r = \partial E_r/\partial\xi$. To be specific, we consider a driving laser pulse in the following profile:

$$a_L(r,\xi) = \begin{cases} a_{L0}\exp(-r^2/r_L^2)\sin(\pi\xi/l_L), & \text{for } 0 \le \xi \le l_L; \\ 0, & \text{otherwise,} \end{cases} \tag{39}$$

where l_L and r_L are the pulse length and the spot size, respectively. Inserting Eq. (39) into the solutions Eqs. (37) and (38), the density perturbation and the axial wakefield in the laser pulse location, $0 \le \xi \le l_L$, are given by

$$\frac{n^{(2)}(r,\xi)}{n_0} = \frac{a_{L0}^2\pi^2}{4\pi^2 - \tilde{k}_{p0}^2 l_L^2}\left\{\cos\tilde{k}_{p0}(\xi - l_L) - \cos\left(\frac{2\pi\xi}{l_L}\right) + \frac{8}{\tilde{k}_{p0}^2 r_L^2}\left(1 - \frac{2r^2}{r_L^2}\right)\times\left\{\cos\tilde{k}_{p0}(\xi - l_L) - 1 - \frac{\tilde{k}_{p0}^2 l_L^2}{4\pi^2}\times\left[\cos\left(\frac{2\pi\xi}{l_L}\right) - 1\right]\right\}\right\}\exp\left(-\frac{2r^2}{r_L^2}\right), \tag{40}$$

and

$$E_z(r,\xi) = \frac{mc^2}{|e|}\frac{k_{p0}^2}{\tilde{k}_{p0}}\frac{a_{L0}^2\pi^2}{(4\pi^2 - \tilde{k}_{p0}^2 l_L^2)}\left[\sin\tilde{k}_{p0}(l_L - \xi) + \frac{\tilde{k}_{p0} l_L}{2\pi}\sin\frac{2\pi\xi}{l_L}\right]\exp\left(\frac{-2r^2}{r_L^2}\right). \tag{41}$$

The transverse wakefield is easily calculated from Eq. (41) by the relation $\partial E_r/\partial z = \partial E_z/\partial r$. When the laser pulse length is nearly equal to the effective plasma wavelength, $l_L = \tilde{\lambda}_{p0} = 2\pi/\tilde{k}_{p0}$, Eq. (40) is reduced to

$$\frac{n^{(2)}(r,\xi)}{n_0} = -\frac{\pi}{4}a_{L0}^2\left[1 + \frac{8}{k_{p0}^2 r_L^2}\left(1 - \frac{2r^2}{r_L^2}\right)\right]\times\exp\left(-\frac{2r^2}{r_L^2}\right)\sin(k_{p0}\xi) + \frac{\delta n}{n_0}, \tag{42}$$

where the corrected term of quantum effects is

$$\frac{\delta n}{n_0} = -\frac{\pi}{8}\Delta a_{L0}^2 k_{p0}\xi\exp\left(-\frac{2r^2}{r_L^2}\right)\cos(k_{p0}\xi) + \frac{\pi}{4}\Delta a_{L0}^2\frac{8}{k_{p0}^2 r_L^2}\times\left(1 - \frac{2r^2}{r_L^2}\right)\exp\left(-\frac{2r^2}{r_L^2}\right)\sin(k_{p0}\xi). \tag{43}$$

The first term on the right of Eq. (42) is the classical density perturbation [25,26], which is shown in Fig. 1, whereas the second term is the contribution of quantum effects to the perturbation of electron number densities, which is represented in Fig. 2. When taking $l_L = \tilde{\lambda}_{p0}$, the axial wakefield in Eq. (41) is maximum and the wakefield within the laser pulse is written by

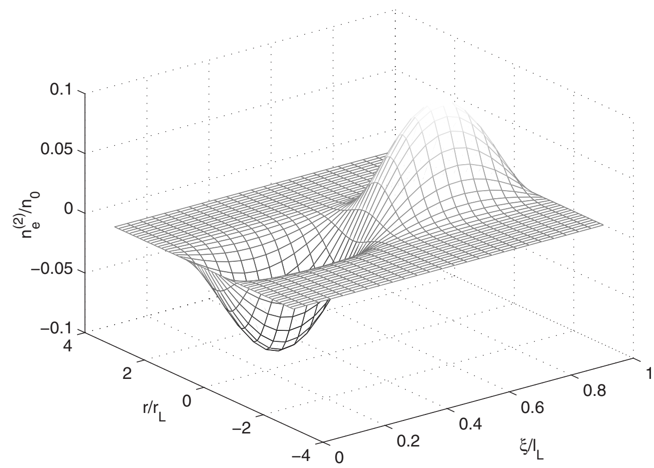

FIG. 1. The density wake for $n_e = 5\times10^{19}$ cm^{-3}, $T = 10^3$ K, $l_L = \lambda_{p0} = 4.72\times10^{-4}$ cm, $a_{L0}^2 = 0.1$, and $r_L = 8\times10^{-4}$ cm. The laser pulse extends over the region $0 \le (z-ct)/l_L \le 1$.

$$E_z(r,\xi) = -\frac{mc^2\pi a_{L0}^2}{4|e|}k_{p0}\exp\left(\frac{-2r^2}{r_L^2}\right)\left[\left(1 - \frac{\xi}{l_L}\right)\cos(k_{p0}\xi) + (2\pi)^{-1}\sin(k_{p0}\xi)\right] + \delta E_z, \tag{44}$$

where the corrected term of quantum effects is

$$\delta E_z = \frac{\Delta}{2}\frac{mc^2\pi a_{L0}^2}{4|e|}k_{p0}\exp\left(\frac{-2r^2}{r_L^2}\right)\left[\left(1 - \frac{\xi}{l_L}\right)\cos(k_{p0}\xi) + (2\pi)^{-1}\sin(k_{p0}\xi)\right] + \frac{\Delta}{2}\frac{mc^2\pi a_{L0}^2}{4|e|}k_{p0}^2\xi\exp\left(\frac{-2r^2}{r_L^2}\right)\times\left[\left(1 - \frac{\xi}{l_L}\right)\sin(k_{p0}\xi) - (2\pi)^{-1}\cos(k_{p0}\xi)\right]. \tag{45}$$

The first term on the right of Eq. (44) is the axial wakefield without considering quantum effects [25,26], which is shown

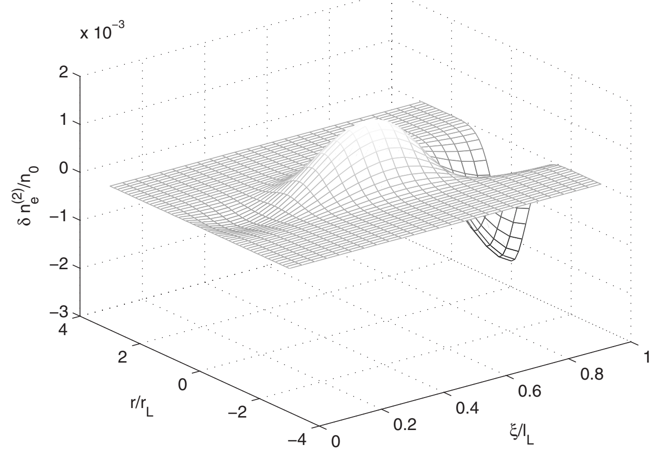

FIG. 2. The modification of density wake for $n_e = 5\times10^{19}$ cm^{-3}, $T = 10^3$ K, $l_L = \tilde{\lambda}_{p0} = 4.68\times10^{-4}$ cm, $a_{L0}^2 = 0.1$, and $r_L = 8\times10^{-4}$ cm. The laser pulse extends over the region $0 \le (z-ct)/l_L \le 1$.

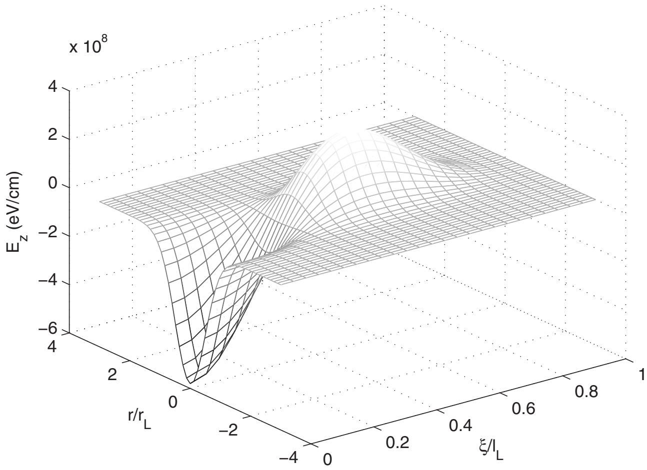

FIG. 3. The axial wakefield for $n_e=5\times10^{19}\ \mathrm{cm}^{-3}$, $T=10^3$ K, $l_L=\lambda_{p0}=4.72\times10^{-4}$ cm, $a_{L0}^2=0.1$, and $r_L=8\times10^{-4}$ cm. The laser pulse extends over the region $0\le(z-ct)/l_L\le1$.

in Fig. 3, whereas the second term is the contribution of quantum effects to the laser wakefield, which is represented in Fig. 4. Through comparing Figs. 1 and 3 with Figs. 2 and 4, we can see that quantum corrections to the electron number densities and the electric field of the laser wakefield cannot be neglected. Compared with the classical density perturbation, the phases of the first term and the second term on the right of Eq. (43) shift $\pi/2$ and π, respectively. It indicates that quantum effects weaken laser wakefield and the accelerating field of the laser wakefield, which is a classical manifestation of quantum decoherence.

IV. DISCUSSION AND CONCLUSION FOR QUANTUM EFFECTS

In this section, we will examine the correction of the quantum effects to the plasma frequency. Equations (35) and

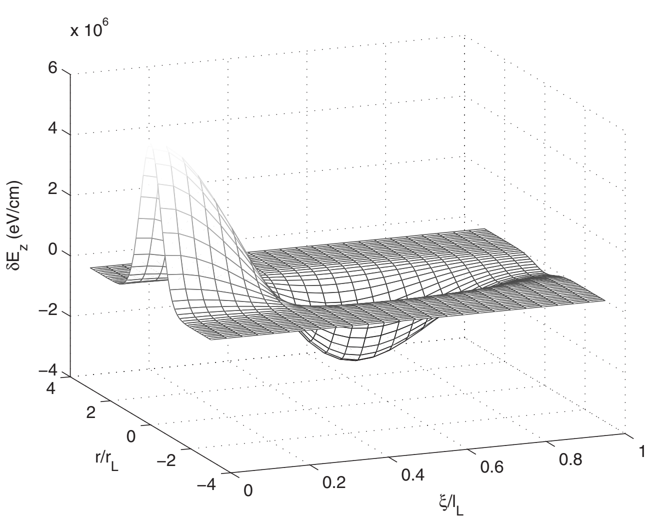

FIG. 4. The modification of axial wakefield for $n_e=5\times10^{19}\ \mathrm{cm}^{-3}$, $T=10^3$ K, $l_L=\tilde{\lambda}_{p0}=4.68\times10^{-4}$ cm, $a_{L0}^2=0.1$, and $r_L=8\times10^{-4}$ cm. The laser pulse extends over the region $0\le(z-ct)/l_L\le1$.

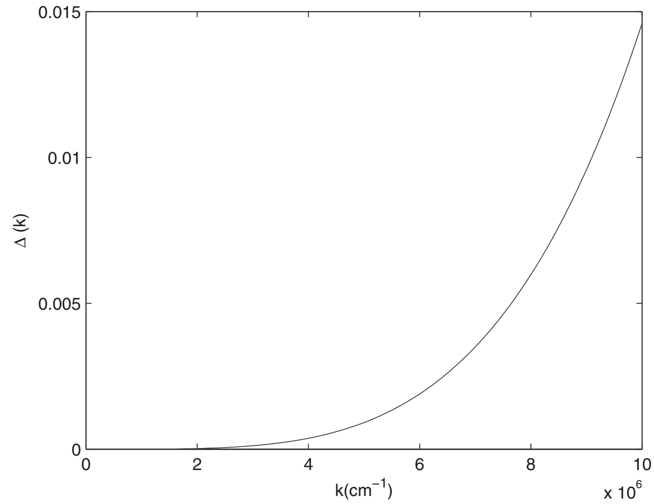

FIG. 5. The corrected term of quantum effects $\Delta(k)$ for different values of the wave number k. The electric equilibrium number density $n_0=5\times10^{19}\ \mathrm{cm}^{-3}$, temperature $T=10^3$ K.

(36) show that quantum effects enlarge the frequency of plasmas and it can be explained by introducing effective charge $e_{(eff)}=\sqrt{(1+\Delta)}e$ and effective electron number density $n_{(eff)}=(1+\Delta)n_0$, i.e.,

$$\tilde{\omega}_{p0}^2=(1+\Delta)\omega_{p0}^2=\frac{4\pi e_{(eff)}^2 n_0}{m_e}=\frac{4\pi e^2 n_{(eff)}}{m_e}. \tag{46}$$

Quantum effects induce effective charge or effective number density of electrons, and the Debye length is found to be reduced. Thus, quantum effects play a role of screening effect for plasma electrons here. From the expression of relativistic quantum corrected term

$$\Delta=\frac{k^4}{4k_c^2k_{p0}^2}(1-\beta), \tag{47}$$

we can obviously find that the contribution of quantum effects is weaken by relativistic effects.

The corrected term is estimated by taking laboratory values: electron number density $n_0=5\times10^{19}\ \mathrm{cm}^{-3}$, electron temperature $T=10^3$ K. Some characteristic parameters of plasmas are calculated as follows: $\omega_{p0}=3.99\times10^{14}\ \mathrm{s}^{-1}$, $k_{p0}=1.33\times10^4\ \mathrm{cm}^{-1}$, $\lambda_{p0}=4.72\times10^{-4}$ cm, $\lambda_B=\hbar/mv_{th}=6.65\times10^{-8}$ cm, $\lambda_F=8.77\times10^{-8}$ cm, and $\lambda_D=4.36\times10^{-8}$ cm. Apparently, the electron thermal de Broglie wavelength approaches the electron Fermi wavelength λ_F and exceeds the electron Debye radius, so the quantum effects cannot be neglected under the above regime of plasmas. It is calculated that the corrected term of quantum effects is $\Delta\sim10^{-2}$ when the wave number of the electron plasma wave is $k=9.09\times10^6\ \mathrm{cm}^{-1}$ and $\beta\sim a_L\approx0.3$. The contribution of the quantum effects to the effective plasma frequency $\Delta(k)$ is displayed in Fig. 5.

To summarize, by using a relativistic quantum hydrodynamic model, the influence of quantum effects on the interaction of intense laser field with plasmas is investigated. Relativistic hydrodynamic equations are derived by using the covariant Wigner function and Dirac equation. Based on the relativistic quantum hydrodynamic equations and Poisson equation, the perturbation of the electron number densities and the electric field of the laser wakefields containing quantum effects are deduced. The quantum corrections to the plasma frequency are examined and estimated by taking the parameters of laboratory plasmas. Results show that quantum effects suppress the perturbation of the electron number densities and the electric field of the laser wakefields. In other words, quantum effects weaken the laser wakefields and the accelerating field of laser wakefields, which is a classical manifestation of quantum decoherence. The derived correction can be explained by an additional effective pressure created in plasmas by quantum fluctuations. The additional pressure leads to more dispersive plasma wave, which weakens the laser wakefields. Our analysis reveals that quantum effects enlarge the frequency of plasma via a screening process and it is equivalent to increase the effective charge of plasma electrons. The contribution of quantum effects is weaken by relativistic effects.

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